

New predictions on the mass of the 1^{-+} light hybrid meson from QCD sum rules

Zhuo-Ran Huang¹, Hong-Ying Jin¹ and Zhu-Feng Zhang²

¹Zhejiang Institute of Modern Physics, Zhejiang University, Zhejiang Province, P. R. China

²Physics Department, Ningbo University, Zhejiang Province, P. R. China

We calculate the coefficients of the dimension-8 quark and gluon condensates in the current-current correlator of 1^{-+} light hybrid current $g\bar{q}(x)\gamma_\nu iG_{\mu\nu}(x)q(x)$. With inclusion of these higher-power corrections and updating the input parameters, we re-analyze the mass of the 1^{-+} light hybrid meson from Monte-Carlo based QCD sum rules. Considering the possible violation of factorization of higher dimensional condensates and variation of $\langle g^3 G^3 \rangle$, we obtain a conservative mass range 1.72–2.60 GeV, which favors $\pi_1(2015)$ as a better hybrid candidate compared with $\pi_1(1600)$ and $\pi_1(1400)$.

PACS numbers: 12.38.Lg, 12.39.Mk, 14.40.Rt

I. INTRODUCTION

Mesons with exotic quantum numbers have long been attractive in hadron physics, among which are the $J^{PC} = 1^{-+}$ isovector states $\pi_1(1400)$, $\pi_1(1600)$ and $\pi_1(2015)$ identified in the experiments [1]. The construction of these states are not quite clear, four-quark states [2–5] and hybrid states are most possible explanations. Theoretical studies via different methods have shown that some of these states can be considered as good light hybrid candidates. In the bag model, the predicted mass of 1^{-+} light hybrid meson is around 1.5 GeV [6]; the mass from the flux tube model is found to be in the range 1.7–1.9 GeV [7]; the lattice QCD prediction of 1^{-+} mass is 1.9–2.2 GeV [8]. Calculations based on QCD sum rules [9] have been conducted by different groups [10–15] to NLO of $d \leq 6$ contributions, and the latest versions of the predicted mass are 1.80 ± 0.06 GeV in [16] and 1.71 ± 0.22 GeV in [17]. Although the hybrid explanation for $\pi_1(1600)$ is supported by previous sum rule analyses, the hybrid assignment of $\pi_1(2015)$ is also proposed [16]. Thus the calculation of higher power corrections (HPC) of the OPE is interesting and of value. How and how much the HPC affect the mass prediction would lead to totally different conclusions.

In this paper, we focus on the mass prediction of the 1^{-+} light hybrid meson using QCD sum rule method. We will first present our calculation of the coefficients of dimension-8 condensates and then include these higher dimensional contributions in the numerical analysis. Due to the possible violation of factorization of $d = 6-8$ condensates and variation of $\langle g^3 G^3 \rangle$ condensate, we will consider a conservative range of the mass prediction. We shall compare the results in $d \leq 8$ case with those in $d \leq 6$ case to show the variation of the mass prediction with inclusion of dimension-8 contributions. In order to obtain an objective conclusion, we shall pay special attention to the fixing of the continuum threshold s_0 , which is not rigorously constrained in the original SVZ sum rules and therefore cause uncertainties. To solve the problem, some authors use the stability criterion to fix s_0 [13, 16]. In this work, we shall fit the sum rules following the matching procedure introduced by Leinweber in [18] and successfully performed in some other works [19–22], from which the continuum threshold s_0 is an output parameter and an uncertainty analysis can be provided. For the explicit consideration of higher power corrections is not seen very often in previous sum rule calculations, we will give a slightly more detailed presentation of our calculation and analysis.

II. OPE FOR THE CURRENT-CURRENT CORRELATOR

We start from the two-point correlator

$$\begin{aligned}\Pi_{\mu\nu}(q^2) &= i \int d^4x e^{iqx} \langle 0 | T [j_\mu(x) j_\nu^\dagger(0)] | 0 \rangle \\ &= (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi_v(q^2) + q_\mu q_\nu \Pi_s(q^2)\end{aligned}\quad (1)$$

where $j_\mu(x) = g\bar{q}(x)\gamma_\nu iG_{\mu\nu}(x)q(x)$, and the invariants $\Pi_v(q^2)$ and $\Pi_s(q^2)$ correspond respectively to 1^{-+} and 0^{++} contributions.

The correlator obeys the standard dispersion relation

$$\Pi_{v/s}(q^2) = \frac{1}{\pi} \int_0^\infty ds \frac{\text{Im}\Pi_{v/s}(s)}{s - q^2 - i\epsilon}.\quad (2)$$

In this paper, we focus on the dimension-8 corrections to the 1^{-+} mass. Before showing the higher power results we need to mention that coefficients of dimension-8 quark-related operators of the 1^{-+} light hybrid two-point correlator have been calculated in [10] and [11]. In [10] there is only a factorized form of the total result and a complete result is given in [11]. We obtain a new complete result which is consistent with the former factorized form but different from the latter one.

As for dimension-8 gluon operators, there arise IR divergences in the calculation of the quark loops as the result of setting $m_q = 0$ before calculating the integrals. These IR divergences can be canceled after taking operator mixing into account. This process can partly check the calculation about dimension-8 quark and gluon operators and modify the finite part of the coefficients of gluon condensates. Some good examples for the case of $\bar{q}q$ scalar and vector currents are given in [23, 24].

According to the numbers of quark operators in the condensates, dimension-8 quark condensates can be classified into two groups: two-quark $d = 8$ condensates and four-quark $d = 8$ condensates. Only the formers can be mixed to $d = 8$ gluon condensates in LO. We use the dimensional regularization in $n = 4 - \epsilon$ space-time dimensions, thus the $O(\epsilon)$ terms of the two-quark $d = 8$ condensates can be obtained, which are needed to be multiplied by the $\frac{1}{\epsilon}$ subtractions to modify the finite part of the quark loop calculations (see Eq.(5)).

The dimension-8 quark contributions (corresponding to Feynman diagrams in Figure 1) are listed in Appendix A.

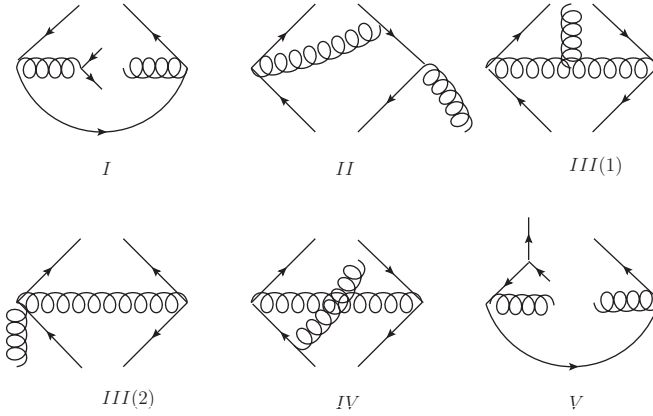


FIG. 1: Feynman diagrams of dimension-8 quark contributions.

Dimension-8 contributions of gluon condensates come from the calculations of quark loops. Here we give the quark propagator up to term $O(q^{-5})$ needed in the calculation of the quark loops:

$$S(q) = S_0(q) + \frac{ig}{2} G_{\rho\mu} S_0(q) \gamma_\mu \frac{\partial}{\partial q_\rho} S_0(q) + \frac{g}{3} D_\alpha G_{\rho\mu} S_0(q) \gamma_\mu \frac{\partial}{\partial q_\alpha} \frac{\partial}{\partial q_\rho} S_0(q) \quad (3)$$

$$- \frac{ig}{8} D_{\alpha 1} D_{\alpha 2} G_{\rho\mu} S_0(q) \gamma_\mu \frac{\partial}{\partial q_{\alpha 1}} \frac{\partial}{\partial q_{\alpha 2}} \frac{\partial}{\partial q_\rho} S_0(q) - \frac{g^2}{4} G_{\rho\mu} G_{\sigma\nu} S_0(q) \gamma_\mu \frac{\partial}{\partial q_\rho} \left[S_0(q) \gamma_\nu \frac{\partial}{\partial q_\sigma} S_0(q) \right],$$

where $D_\mu = \partial_\mu - igA_\mu$ and $S_0(q) = \frac{1}{q}$.

For a massless quark Eq.(3) can be rewritten as

$$S(q) = \frac{\not{q}}{q^2} + \frac{1}{q^4} g q_\alpha \tilde{G}_{\alpha\beta} \gamma_\beta \gamma_5 \quad (4)$$

$$+ \frac{1}{q^6} \left[-\frac{2}{3} g (q_\alpha q_\rho D_\rho G_{\alpha\beta} \gamma_\beta - J_\mu q_\mu \not{q} + q^2 \not{q}) + 2ig q_\alpha q_\rho D_\rho \tilde{G}_{\alpha\beta} \gamma_\beta \gamma_5 \right]$$

$$+ \frac{1}{q^8} \{ -2ig q_\gamma D_\gamma (q^2 \not{q} - q_\mu J_\mu \not{q}) + [-4g (q_\gamma D_\gamma)^2 + g q^2 D^2] q_\alpha \tilde{G}_{\alpha\beta} \gamma_\beta \gamma_5 + 2ig (q_\gamma D_\gamma)^2 q_\alpha G_{\mu\alpha} \gamma_\mu$$

$$+ 2g^2 q_\mu q_\alpha G_{\mu\rho} G_{\alpha\rho} \not{q} + 2g^2 q^2 q_\mu G_{\alpha\rho} G_{\rho\mu} \gamma_\alpha + ig^2 q^2 q_\alpha (\tilde{G}_{\mu\beta} G_{\alpha\beta} - G_{\mu\beta} \tilde{G}_{\alpha\beta}) \gamma_\mu \gamma_5 \},$$

where $\tilde{G}_{\alpha\beta} = \frac{1}{2} \varepsilon_{\alpha\beta\mu\nu} G_{\mu\nu}$, $\gamma_5 = -\frac{i}{4} \varepsilon_{\alpha\beta\mu\nu} \gamma_\alpha \gamma_\beta \gamma_\mu \gamma_\nu$, $J_\mu = D_\nu G_{\mu\nu} = g \sum_{uds} \bar{\psi} \gamma_\mu T^a \psi T^a$. Eq.(4) can also be seen in [25] and [24], but the last term of (4) is missed in [25] and not consistent with [24]. We use (3) rather than (4) in practical calculations for (3) is more convenient in program calculations.

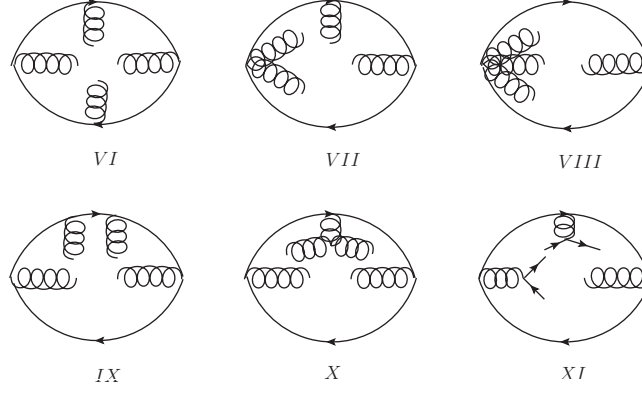


FIG. 2: Feynman diagrams of dimension-8 gluon contributions.

TABLE I: The independent $d = 8$ two-quark condensates and coefficients. $(\gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma)_-$ and $(\gamma_\mu \gamma_\rho \gamma_\sigma)_-$ are totally anti-symmetric tensors.

j	Q_j	C_j^V	D_j^V	C_j^S	D_j^S
1	$-ig^2 \bar{q} [\not{D} G_{\mu\nu}, G_{\rho\sigma}] (\gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma) - q$	$\frac{1}{72}$	$\frac{1}{96}$	$\frac{1}{24}$	$\frac{5}{288}$
2	$-ig^2 \bar{q} [\not{D} G_{\mu\nu}, G_{\mu\nu}] q$	$-\frac{1}{36}$	$-\frac{1}{48}$	$-\frac{1}{12}$	$-\frac{5}{144}$
3	$-ig^2 \bar{q} [G_{\mu\nu}, G_{\rho\sigma}] D_\nu (\gamma_\mu \gamma_\rho \gamma_\sigma) - q$	$\frac{1}{18}$	$\frac{1}{24}$	$\frac{1}{6}$	$\frac{5}{72}$
4	$-ig^2 \bar{q} \{G_{\mu\rho}, G_{\mu\nu}\} \gamma_\nu D_\rho q$	$-\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{3}$	$\frac{3}{36}$
5	$-ig^2 \bar{q} \{D_\mu G_{\nu\mu}, G_{\rho\sigma}\} (\gamma_\nu \gamma_\rho \gamma_\sigma) - q$	$-\frac{1}{9}$	$-\frac{5}{144}$	$-\frac{1}{12}$	$-\frac{1}{18}$
6	$-ig^2 \bar{q} [D_\mu G_{\nu\mu}, G_{\nu\alpha}] \gamma_\alpha q$	$-\frac{1}{18}$	$-\frac{1}{24}$	$-\frac{1}{6}$	$-\frac{5}{72}$
7	$-ig^2 \bar{q} D^2 D_\nu G_{\alpha\nu} \gamma_\alpha q$	0	0	0	0

Gluon contributions from calculations of quark loops (the corresponding Feynman diagrams are depicted in Figure 2) are listed in Appendix A.

The two-quark $d = 8$ condensates in (15) and (19) can be expanded in the basis $\{Q_j\}$ listed in Table I, using the equations of motion and charge conjugation transformation and setting $m_q = 0$. And we also list the expanding coefficients in Table I and the mixing coefficients of quark condensates with gluon condensates in Table II [23]. After

TABLE II: The mixing coefficients in (5) [23].

j	2	3	3	3	3	3	3	4	4	4	6	7
i	6	1	2	3	4	5	6	1	3	4	5	7
Z_i^j	-6	6	-6	-12	12	-6	-3	-6	12	12	6	-6

taking operator mixing into account, the corrective terms to gluon contributions are obtained:

$$\begin{aligned}
\Pi_c^G(q) &= (q_\mu q_\nu - q^2 g_{\mu\nu}) \sum_{j=1}^7 (C_j^V + \epsilon D_j^V) \sum_{i=1}^7 Z_i^j g^{n_i} O_i / (72\pi^2 \omega) \frac{1}{q^4} \\
&+ q_\mu q_\nu \sum_{j=1}^7 (C_j^S + \epsilon D_j^S) \sum_{i=1}^7 Z_i^j g^{n_i} O_i / (72\pi^2 \omega) \frac{1}{q^4} \\
&= (q_\mu q_\nu - q^2 g_{\mu\nu}) \left[\left(-\frac{5}{864\pi^2} + \frac{1}{72\pi^2} \frac{1}{\omega} \right) g^4 O_1 + \left(-\frac{1}{288\pi^2} - \frac{1}{216\pi^2} \frac{1}{\omega} \right) g^4 O_2 \right. \\
&+ \left(\frac{5}{432\pi^2} - \frac{1}{36\pi^2} \frac{1}{\omega} \right) g^4 O_3 + \left(\frac{11}{432\pi^2} - \frac{1}{108\pi^2} \frac{1}{\omega} \right) g^4 O_4 + \left(-\frac{1}{144\pi^2} - \frac{1}{108\pi^2} \frac{1}{\omega} \right) g^3 O_5 \Big] \frac{1}{q^4} \\
&+ q_\mu q_\nu \left[\left(-\frac{1}{96\pi^2} - \frac{1}{24\pi^2} \frac{1}{\omega} \right) g^4 O_1 + \left(-\frac{5}{864\pi^2} - \frac{1}{72\pi^2} \frac{1}{\omega} \right) g^4 O_2 \right. \\
&+ \left. \left(\frac{1}{48\pi^2} + \frac{1}{12\pi^2} \frac{1}{\omega} \right) g^4 O_3 + \left(\frac{19}{432\pi^2} + \frac{5}{36\pi^2} \frac{1}{\omega} \right) g^4 O_4 + \left(-\frac{5}{432\pi^2} - \frac{1}{36\pi^2} \frac{1}{\omega} \right) g^3 O_5 \right] \frac{1}{q^4},
\end{aligned} \tag{5}$$

where $\frac{1}{\omega} = \frac{1}{\epsilon} + \frac{1}{2} \ln 4\pi - \frac{\gamma_E}{2}$ and by which the IR divergences in (24), (25) and (26) can be canceled. Thus the total dimension-8 contributions are the sum of (5) and (20)–(26):

$$\begin{aligned}
\Pi_{\mu\nu}^{d=8}(q^2) &= (q_\mu q_\nu - q^2 g_{\mu\nu}) \left[-\frac{1}{24} g^3 \langle \bar{q}q \rangle \langle \bar{q}Gq \rangle \right. \\
&+ \left(-\frac{1}{108\pi^2} + \frac{1}{144\pi^2} \ln \frac{-q^2}{\mu^2} \right) g^4 O_1 + \left(\frac{1}{216\pi^2} - \frac{1}{432\pi^2} \ln \frac{-q^2}{\mu^2} \right) g^4 O_2 \\
&+ \left(-\frac{1}{108\pi^2} - \frac{1}{72\pi^2} \ln \frac{-q^2}{\mu^2} \right) g^4 O_3 + \left(\frac{1}{54\pi^2} - \frac{1}{216\pi^2} \ln \frac{-q^2}{\mu^2} \right) g^4 O_4 \\
&+ \left(\frac{1}{864\pi^2} - \frac{1}{216\pi^2} \ln \frac{-q^2}{\mu^2} \right) g^3 O_5 + \frac{1}{288\pi^2} g^2 O_7 + \frac{1}{288\pi^2} g^3 O_8 \Big] \frac{1}{q^4} \\
&+ q_\mu q_\nu \left[-\frac{11}{27} g^3 \langle \bar{q}q \rangle \langle \bar{q}Gq \rangle \right. \\
&+ \left(\frac{7}{576\pi^2} - \frac{1}{48\pi^2} \ln \frac{-q^2}{\mu^2} \right) g^4 O_1 + \left(-\frac{7}{516\pi^2} - \frac{1}{144\pi^2} \ln \frac{-q^2}{\mu^2} \right) g^4 O_2 \\
&+ \left(-\frac{13}{288\pi^2} + \frac{1}{24\pi^2} \ln \frac{-q^2}{\mu^2} \right) g^4 O_3 + \left(-\frac{7}{288\pi^2} + \frac{5}{72\pi^2} \ln \frac{-q^2}{\mu^2} \right) g^4 O_4 \\
&+ \left. \left(\frac{11}{576\pi^2} - \frac{1}{72\pi^2} \ln \frac{-q^2}{\mu^2} \right) g^3 O_5 + \frac{1}{192\pi^2} g^2 O_7 \right] \frac{1}{q^4},
\end{aligned} \tag{6}$$

where $\langle \bar{q}Gq \rangle = \langle \bar{q} \frac{\lambda^a}{2} G_{\mu\nu}^a \sigma_{\mu\nu} q \rangle$, $\sigma_{\mu\nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu]$.

Notice that the quark condensates have been factorized in (6) so as to conduct the sum rule analysis. As is well-known, factorization hypothesis may have large uncertainties as observed in some other channels [27–32]. Therefore we shall consider the possible violation of factorization for quark condensates in the numerical analysis. As for the values of $\langle G^4 \rangle$ condensates ($O_1 - O_4$), one may also think of using factorization. However, for reasons in [33], factorization hypothesis may as well not be reliable in $\langle G^4 \rangle$ case. Therefore we choose to use a modified factorization proposed in [34] and supported in [36], which suggests an overestimation of factorization and are based on two technologies: factorization of quartic heavy quark condensates and heavy quark expansion. In the framework of this modified factorization, $O_1 - O_4$ can be expressed in terms of the condensate $\phi = \text{Tr} G_{\nu\mu} G_{\mu\rho} \text{Tr} G_{\nu\tau} G_{\tau\rho}$, which has been clarified in [34] to reasonably satisfy the factorization approximation. Thus after fitting ϕ using factorization, $O_1 - O_4$ can be estimated as follows

$$\begin{aligned}
g^4 O_1 &= \frac{1}{12} \langle g^2 G^2 \rangle^2, \quad g^4 O_2 = -\frac{5}{48} \langle g^2 G^2 \rangle^2 + 2g^4 \phi, \\
g^4 O_3 &= g^4 O_4 = -\frac{1}{192} \langle g^2 G^2 \rangle^2 + \frac{1}{2} g^4 \phi.
\end{aligned} \tag{7}$$

With regard to other $d = 8$ gluon condensates, a scale $M^2 \approx 0.3 \text{ GeV}^2$ is estimated in [35, 36], which characterizes

the average off-shellness of the vacuum gluons and quarks:

$$g^3 O_5 = -\frac{3}{2}g^4 \langle \bar{q}q \rangle^2 M^2, g^2 O_7 = -\frac{4}{3}g^4 \langle \bar{q}q \rangle^2 M^2, g^3 O_8 = \langle g^3 G^3 \rangle M^2, \quad (8)$$

and we shall also consider the violation of factorization of O_5 and O_7 in the matching procedure.

III. QCD SUM RULES FOR THE 1^{-+} LIGHT HYBRID MESON

The $d \leq 6$ contributions to $\Pi_v(q^2)$ including the NLO corrections to the perturbative and the $\langle \alpha_s G^2 \rangle$ and $\alpha_s \langle \bar{q}q \rangle^2$ terms can be found in [10–15]. $\Pi_v^{d \leq 6}(q^2)$ can be written as

$$\Pi_v^{d \leq 6}(q^2) = a_{11} q^4 \ln \frac{-q^2}{\mu^2} + a_{12} q^4 \ln^2 \frac{-q^2}{\mu^2} + b_{11} \ln \frac{-q^2}{\mu^2} + b_{12} \ln^2 \frac{-q^2}{\mu^2} + c_{11} \frac{1}{-q^2} + c_{12} \frac{1}{-q^2} \ln \frac{-q^2}{\mu^2} \quad (9)$$

with

$$\begin{aligned} a_{11} &= -\frac{\alpha_s(\mu)}{240\pi^3} \left(1 + \frac{1301}{240} \frac{\alpha_s(\mu)}{\pi} \right), \quad a_{12} = \frac{\alpha_s(\mu)}{240\pi^3} \frac{17}{72} \frac{\alpha_s(\mu)}{\pi}, \\ b_{11} &= -\frac{1}{36\pi} \langle \alpha_s G^2 \rangle \left(1 - \frac{145}{72} \frac{\alpha_s(\mu)}{\pi} \right) - \frac{2}{9} \frac{\alpha_s(\mu)}{\pi} \langle m_q \bar{q}q \rangle, \\ b_{12} &= -\frac{1}{36\pi} \langle \alpha_s G^2 \rangle \frac{8}{9} \frac{\alpha_s(\mu)}{\pi}, \\ c_{11} &= -\frac{4\pi}{9} k_1 \alpha_s \langle \bar{q}q \rangle^2 \left(1 + \frac{1}{108} \frac{\alpha_s(\mu)}{\pi} \right) - \frac{1}{192\pi^2} \langle g^3 G^3 \rangle, \\ c_{12} &= -\frac{4\pi}{9} k_1 \alpha_s \langle \bar{q}q \rangle^2 \frac{47}{72} \frac{\alpha_s(\mu)}{\pi}, \end{aligned}$$

where $\alpha_s(\mu) = 4\pi/(9 \ln(\mu^2/\Lambda_{\text{QCD}}^2))$ is the running coupling constant for three flavors, and k_1 indicates the deviation from vacuum saturation of $d = 6$ quark condensates.

In addition, $\Pi_v^{d=8}(q^2)$ can be obtained from (6), (7) and (8):

$$\Pi_v^{d=8}(q^2) = d_{11} \frac{1}{q^4} + d_{12} \frac{1}{q^4} \ln \frac{-q^2}{\mu^2} \quad (10)$$

with

$$\begin{aligned} d_{11} &= -\frac{\pi}{6} k_2 \alpha_s(\mu) \langle \bar{q}q \rangle \langle g \bar{q} G q \rangle - \frac{1}{216} \langle \alpha_s G^2 \rangle^2 - \frac{11}{108} k_2 \alpha_s(\mu) \cdot \alpha_s \langle \bar{q}q \rangle^2 \cdot M^2 + \frac{1}{288\pi^2} \langle g^3 G^3 \rangle M^2, \\ d_{12} &= -\frac{1}{648} \langle \alpha_s G^2 \rangle^2 + \frac{1}{9} k_2 \alpha_s(\mu) \cdot \alpha_s \langle \bar{q}q \rangle^2 \cdot M^2, \end{aligned}$$

where k_2 indicates the deviation from vacuum saturation of $d=8$ condensates.

The Borel transformation of $\Pi_v^{\text{OPE}}(q^2)$ can be written as

$$\begin{aligned} \Pi_v^{\text{OPE}}(\tau) \equiv \frac{1}{\tau} \hat{B}_\tau \Pi_v^{\text{OPE}}(q^2) &= a_{11} \frac{-2}{\tau^3} + a_{12} \frac{2}{\tau^3} (2\gamma_E - 3 + 2 \ln(\tau\mu^2)) + b_{11} \frac{-1}{\tau} + b_{12} \frac{2}{\tau} (\gamma_E + \ln(\tau\mu^2)) \\ &+ c_{11} + c_{12} (-\gamma_E - \ln(\tau\mu^2)) + d_{11} \tau + d_{12} \tau (1 - \gamma_E - \ln(\tau\mu^2)). \end{aligned} \quad (11)$$

By using the single narrow resonance spectral density ansatz $\text{Im}\Pi_v^{\text{phen}}(s) = \pi f_H^2 m_H^4 \delta(s - m_H^2) + \text{Im}\Pi_v^{\text{OPE}}(s) \theta(s - s_0)$, where s_0 is the continuum threshold, f_H and m_H denote the coupling of the hadron to the current and the mass of the hadron respectively, we can obtain the phenomenological representation of $\Pi_v^{\text{phen}}(\tau, s_0, f_H, m_H)$ via the dispersion relation:

$$\Pi_v^{\text{phen}}(\tau, s_0, f_H, m_H) = \frac{1}{\pi} \int_0^\infty \text{Im}\Pi_v^{\text{phen}}(s) e^{-s\tau} ds. \quad (12)$$

Then the master equation for QCDSR can be written as

$$\Pi_v^{\text{OPE}}(\tau) = \Pi_v^{\text{phen}}(\tau, s_0, f_H, m_H), \quad (13)$$

physical properties of the relevant hadron, i.e., m_H , f_H and s_0 , should satisfy Eq.(13).

In order to present the influence of the $d = 8$ contributions, we will conduct the sum rule analysis both in $d \leq 6$ and $d \leq 8$ cases. Before those, we should clarify our criteria for establishing the sum rule window in which the mass prediction is reliable. On the OPE side, we wish the Borel parameter τ is as small as possible so that power series converge as quickly as possible. On the hadron spectrum side, our wish is the opposite, because a larger τ can better suppress contributions of the excited states and continuum. The common procedure without considering the higher power contributions is usually as follows: 1. keep the highest dimensional contributions (HDC, normally dimension-6 contributions) no more than 10% (or 15%) of the total OPE contributions to ensure the convergence of OPE, which gives the upper bound of τ ; 2. make sure that the contributions from the continuum are under 50% of the total contributions, which ensures the validity of the narrow resonance ansatz and gives the lower bound of τ . For our case, if we require dimension-8 contributions are less than 15 percent, it means we choose a window with a larger upper bound compared with $d \leq 6$ case. This choice enhances suppression of excited states and continuum, but the convergence of OPE gets worse, which increases the uncertainties of the OPE side. On the other hand, if we still require the dimension-6 contributions are less than 15 percent, uncertainties from the truncation of OPE are indeed decreased (because the dimension-8 contributions are now taken into account), but the validity of the narrow resonance ansatz is not improved. Apparently, to keep a balance should be a good resolution. Our choice is that make sure both $1\% < d = 8$ contributions $< 5\%$ and $20\% < d = 6$ contributions $< 35\%$ (correspondingly the perturbative and $d < 6$ contributions would be totally 120%–140% because the signs of the $d = 6$ and $d = 8$ contributions are minus), which ensures the OPE series converge in a proper trend and also a larger upper bound of τ is obtained compared with $d \leq 6$ case, thus uncertainties from both sides of the master equation are reduced.

In the original SVZ sum rules, the continuum threshold s_0 cannot be rigorously constrained. To overcome this shortcoming and make our conclusion more reliable, we use a weighted-least-square method following Leinweber [18] to match the two sides of Eq.(13) in the sum rule window.

By randomly generating 200 sets of Gaussian distributed phenomenological input parameters with given uncertainties (10% uncertainties, which are typical uncertainties in QCDSR) at $\tau_j = \tau_{\min} + (\tau_{\max} - \tau_{\min}) \times (j - 1)/(n_B - 1)$, where $n_B = 21$, we can estimate the standard deviation $\sigma_{\text{OPE}}(\tau_j)$ for $\Pi_v^{\text{OPE}}(\tau_j)$. Then, the phenomenological output parameters s_0 , f_H and m_H can be obtained by minimizing

$$\chi^2 = \sum_{j=1}^{n_B} \frac{(\Pi^{\text{OPE}}(\tau_j) - \Pi^{\text{phen}}(\tau_j, s_0, f_H, m_H))^2}{\sigma_{\text{OPE}}^2(\tau_j)}. \quad (14)$$

We use two sets of parameters as the central values of inputs (see Table III) to conduct the matching procedures respectively. Values in set I are from a recent review of QCD sum rules [37]. We choose this set of values to avoid subjective factors in choosing the inputs. We also notice that the value of $g^3 \langle G^3 \rangle$ in [37] is different from the previous one used in [15–17]. This value changes from $1.2 \text{ GeV}^2 \langle \alpha_s G^2 \rangle$ (from dilute gas instantons [38] and lattice calculations [39]) to $8.2 \text{ GeV}^2 \langle \alpha_s G^2 \rangle$ (from charmonium systems [36]), which largely affects the mass predictions. To make our conclusions more reliable and to provide a comparison of the $d \leq 6$ results in this work and those from previous analyses, we maintain the value of $g^3 \langle G^3 \rangle$ the small one in set II.

As in our previous paper [17], we generate 2000 sets of Gaussian distributed input parameters with 10% uncertainties, and for each set we minimize χ^2 to obtain a set of phenomenological output parameters, after this procedure is finished, we can estimate the uncertainties of s_0 , f_H and m_H .

TABLE III: Different input phenomenological parameters (at scale $\mu_0 = 1 \text{ GeV}$).

	Λ_{QCD}	$\langle \alpha_s G^2 \rangle$	m_q	$\langle g^3 G^3 \rangle$	$\alpha_s \langle \bar{q}q \rangle^2$	$\langle g\bar{q}Gq \rangle$
Set I	0.353 GeV	0.07 GeV^4	0.007 GeV	$8.2 \text{ GeV}^2 \langle \alpha_s G^2 \rangle$	$1.5 \times 10^{-4} \text{ GeV}^4$	$0.8 \text{ GeV}^2 \langle \bar{q}q \rangle$
Set II	0.353 GeV	0.07 GeV^4	0.007 GeV	$1.2 \text{ GeV}^2 \langle \alpha_s G^2 \rangle$	$1.5 \times 10^{-4} \text{ GeV}^4$	$0.8 \text{ GeV}^2 \langle \bar{q}q \rangle$

Finally, before proceeding with numerical calculations, renormalization-group (RG) improvement of the sum rules, i.e., substitutions $\mu^2 \rightarrow 1/\tau$ in Eq.(13), is needed [40]. In addition, the anomalous dimensions for condensate $\langle g^3 G^3 \rangle$ and $\langle \bar{q}q \rangle \langle g\bar{q}Gq \rangle$ also should be implemented by multiplying $\langle g^3 G^3 \rangle$ and $\langle \bar{q}q \rangle \langle g\bar{q}Gq \rangle$ by a factor $L(\mu_0)^{-23/27}$ and $L(\mu_0)^{10/27}$ respectively, where $L(\mu_0) = [\ln(1/(\tau\Lambda_{\text{QCD}}^2))/\ln(\mu_0^2/\Lambda_{\text{QCD}}^2)]$, μ_0 is the renormalization scale for condensates [9, 41]. The coupling constant f_H also should be multiplied by a factor $L(m)^{-32/81}$, f_H then receives its value at hybrid mass shell. In this paper, we neglect the anomalous dimensions for operators $O_1 - O_8$, which are not calculated yet and very likely to have small effects on the mass prediction.

Our matching results with input parameters in Set I and Set II can be seen in Appendix B. We consider violation of factorization by different factors (up to 3 for dimension-6 condensates [27–32], and up to 5 for dimension-8 condensates

[26]). The upper bounds of sum rule windows in each table are obtained by different demands on $|\text{HDC}|/\text{OPE}$. The matching results, including the medians and the asymmetric standard deviations from the medians for s_0 , m_H and f_H^2 , are reported. By inputting Gaussian distributed input parameters with 10% uncertainties, we obtain some Gaussian-like distribution results for s_0 , m_H and f_H^2 with uncertainties $<10\%$, this implies the matching results are very stable with different input parameters. Following our criteria above for establishing the window, the phenomenological outputs in the fourth column of each table are the most reliable (optimal windows) for each case. In fact, we can see that the predictions are not very sensitive to the variation of the range of the window. All output parameters slightly decrease for stronger constraints on contributions from HDC. In addition, we also list the results deduced from $d \leq 6$ contributions in the optimal windows of $d \leq 8$ cases to show the variations of the sum rules in these regions after considering the dimension-8 contributions.

Under the considerations of possible violation of factorization and different values of $\langle g^3 G^3 \rangle$, we obtain a mass range 1.88–2.60 GeV from the optimal windows. Furthermore, We shall also consider effects of tachyonic gluon mass [42–44] beyond the original OPE as in [16]. The lowest order correction due to this effect can be found in [16], which leads to decreases in hybrid mass predictions. Taking this effect into account, the lower bound of the mass range would further decrease, therefore we obtain as conclusion a quite conservative range of the predicted mass, i.e. 1.72–2.60 GeV, which covers $\pi_1(2015)$ and is hard to favor $\pi_1(1400)$ and $\pi_1(1600)$ as hybrids.

As a supplement of our analysis, we also consider as above a conservative mass range in $d \leq 6$ case. With the small $\langle g^3 G^3 \rangle$ value used in [15, 16], the range is 1.55–2.29 GeV, which is consistent with previous predictions within errors and covers $\pi_1(1600)$. In the large $\langle g^3 G^3 \rangle$ case, the predicted range is 1.84–2.46 GeV. Notice that even in this case, the hybrid assignment of $\pi_1(1600)$ can hardly be favored.

More details of the weighted-least-square matching method can be seen in our previous work on the 1^{-+} light hybrid meson [17]. In that work, we concentrate ourselves on the sum rule analysis based on the matching procedure, especially the uncertainty analysis. However, the dimension-8 coefficients used there are not a complete form (only the factorized quark condensates in [10]). And we follow our earlier works [14, 15] in choosing the inputs there and neglect the violation of saturation hypothesis. Moreover, the sum rule window there is just established by keeping $\text{HDC} < 10\%$ as the common procedure, lacking in an explicit consideration of the convergence of OPE. All these lead to the discrepancy of the predictions.

IV. SUMMARY

We have calculated the dimension-8 coefficients of the two-point correlator of the current $g\bar{q}(x)\gamma_\nu iG_{\mu\nu}(x)q(x)$. We find that the inclusion of the dimension-8 condensate contributions in QCDSR analysis increases the predicted mass, and so does the effect of violation of factorization of higher dimensional condensates. Besides, the variation of the value of $\langle g^3 G^3 \rangle$ also have effects on increasing the mass prediction. Therefore all these new effects suggest that the 1^{-+} light hybrid meson may have a larger mass compared with previous QCDSR predictions. From our analysis, the conservative range of the mass is 1.72–2.60 GeV, which covers $\pi_1(2015)$ and disfavors the hybrid explanations for $\pi_1(1600)$ and $\pi_1(1400)$. One can also consider the central value 2.16 GeV in this range as a very crude estimation of the mass.

As for the effect of the dimension-8 contributions in determining the 1^{-+} mass, it's hard to draw a definite conclusion due to the uncertainties from violation of factorization. From the data in Appendix B, we find that 4%–9% underestimation would be led to by neglecting the $d = 8$ condensate contributions in the case of the 1^{-+} light hybrid state.

Acknowledgments

This work is supported by NSFC under grant 11175153, 11205093 and 11347020, and supported by K. C. Wong Magna Fund in Ningbo University.

Appendix A: Results of Calculations of Feynman Diagrams

We list in this appendix the results of the calculations of the Feynman diagrams in Figure 1 and Figure 2.

$$\begin{aligned}
\pi_{\mu\nu}^{\text{I}}(q) = & (q_\mu q_\nu - q^2 g_{\mu\nu}) \left[\left(-\frac{1}{18} - \frac{\epsilon}{24} \right) i g^2 \langle \bar{q} D_\mu G_{\mu\alpha} G_{\nu\beta} \gamma_\alpha \gamma_\nu \gamma_\beta q \rangle \right. \\
& + \left(-\frac{2}{9} + \frac{\epsilon}{36} \right) i g^2 \langle \bar{q} D_\rho G_{\mu\alpha} G_{\mu\beta} \gamma_\alpha \gamma_\rho \gamma_\beta q \rangle + \left(-\frac{1}{18} - \frac{\epsilon}{24} \right) i g^2 \langle \bar{q} D_\nu G_{\mu\alpha} G_{\nu\beta} \gamma_\alpha \gamma_\mu \gamma_\beta q \rangle \left. \right] \frac{1}{q^4} \\
& + q_\mu q_\nu \left[\left(-\frac{1}{6} - \frac{5\epsilon}{72} \right) i g^2 \langle \bar{q} D_\mu G_{\mu\alpha} G_{\nu\beta} \gamma_\alpha \gamma_\nu \gamma_\beta q \rangle + \left(\frac{1}{3} + \frac{\epsilon}{18} \right) i g^2 \langle \bar{q} D_\rho G_{\mu\alpha} G_{\mu\beta} \gamma_\alpha \gamma_\rho \gamma_\beta q \rangle \right. \\
& + \left. \left(-\frac{1}{6} - \frac{5\epsilon}{72} \right) i g^2 \langle \bar{q} D_\nu G_{\mu\alpha} G_{\nu\beta} \gamma_\alpha \gamma_\mu \gamma_\beta q \rangle \right] \frac{1}{q^4},
\end{aligned} \tag{15}$$

$$\begin{aligned}
\pi_{\mu\nu}^{\text{II}}(q) = & (q_\mu q_\nu - q^2 g_{\mu\nu}) \left[\frac{5}{18} i g^3 \langle \bar{q} \gamma_\alpha T^a q \bar{q} T^a G_{\mu\beta} \gamma_\mu \gamma_\alpha \gamma_\beta q \rangle \right. \\
& - \frac{1}{18} i g^3 \langle \bar{q} \gamma_\alpha T^a q \bar{q} T^a G_{\mu\beta} \gamma_\alpha \gamma_\mu \gamma_\beta q \rangle - \frac{2}{9} i g^3 \langle \bar{q} \gamma_\alpha T^a q \bar{q} T^a G_{\alpha\beta} \gamma_\beta q \rangle \left. \right] \frac{1}{q^4} \\
& + q_\mu q_\nu \left[-\frac{1}{6} i g^3 \langle \bar{q} \gamma_\alpha T^a q \bar{q} T^a G_{\mu\beta} \gamma_\mu \gamma_\alpha \gamma_\beta q \rangle \right. \\
& + \left. \frac{5}{6} i g^3 \langle \bar{q} \gamma_\alpha T^a q \bar{q} T^a G_{\mu\beta} \gamma_\alpha \gamma_\mu \gamma_\beta q \rangle - \frac{2}{3} i g^3 \langle \bar{q} \gamma_\alpha T^a q \bar{q} T^a G_{\alpha\beta} \gamma_\beta q \rangle \right] \frac{1}{q^4},
\end{aligned} \tag{16}$$

$$\begin{aligned}
\pi_{\mu\nu}^{\text{III}}(q) = & (q_\mu q_\nu - q^2 g_{\mu\nu}) \left(\frac{1}{12} g^3 \langle f^{abc} G_{\alpha\beta}^a \bar{q} \gamma_\alpha T^b q \bar{q} T^c \gamma_\beta q \rangle \right) \frac{1}{q^4} \\
& + q_\mu q_\nu \left(-\frac{3}{4} g^3 \langle f^{abc} G_{\alpha\beta}^a \bar{q} \gamma_\alpha T^b q \bar{q} T^c \gamma_\beta q \rangle \right) \frac{1}{q^4},
\end{aligned} \tag{17}$$

$$\begin{aligned}
\pi_{\mu\nu}^{\text{IV}}(q) = & (q_\mu q_\nu - q^2 g_{\mu\nu}) \left[\frac{1}{18} g^3 \langle \bar{q} G_{\mu\nu} T^a \sigma_{\mu\nu} \gamma_\alpha q \bar{q} T^a \gamma_\alpha q \rangle \right. \\
& + \frac{1}{36} i g^3 \langle \bar{q} \gamma_\beta T^a q \bar{q} G_{\alpha\beta} T^a \gamma_\alpha q \rangle + \frac{1}{18} g^3 \langle \bar{q} T^a G_{\mu\nu} \gamma_\alpha \sigma_{\mu\nu} q \bar{q} T^a \gamma_\alpha q \rangle \\
& - \frac{1}{36} i g^3 \langle \bar{q} \gamma_\beta T^a q \bar{q} T^a G_{\alpha\beta} \gamma_\alpha q \rangle + \frac{2}{9} g^2 \left\langle \bar{q} \overleftrightarrow{D}_\alpha T^a \gamma_\beta \overleftrightarrow{D}_\alpha q \bar{q} T^a \gamma_\beta q \right\rangle \left. \right] \frac{1}{q^4} \\
& + q_\mu q_\nu \left[\frac{1}{4} g^3 \langle \bar{q} G_{\mu\nu} T^a \sigma_{\mu\nu} \gamma_\alpha q \bar{q} T^a \gamma_\alpha q \rangle - \frac{1}{4} i g^3 \langle \bar{q} \gamma_\beta T^a q \bar{q} G_{\alpha\beta} T^a \gamma_\alpha q \rangle \right. \\
& + \frac{1}{4} g^3 \langle \bar{q} T^a G_{\mu\nu} \gamma_\alpha \sigma_{\mu\nu} q \bar{q} T^a \gamma_\alpha q \rangle + \frac{1}{4} i g^3 \langle \bar{q} \gamma_\beta T^a q \bar{q} T^a G_{\alpha\beta} \gamma_\alpha q \rangle \\
& + \left. g^2 \left\langle \bar{q} \overleftrightarrow{D}_\alpha T^a \gamma_\beta \overleftrightarrow{D}_\alpha q \bar{q} T^a \gamma_\beta q \right\rangle \right] \frac{1}{q^4},
\end{aligned} \tag{18}$$

$$\begin{aligned}
\pi_{\mu\nu}^{\text{V}}(q) = & (q_\mu q_\nu - q^2 g_{\mu\nu}) \left[\left(-\frac{2}{9} + \frac{\epsilon}{36} \right) i g^2 \left\langle \bar{q} \overleftrightarrow{D}_\rho G_{\mu\alpha} G_{\mu\beta} \gamma_\alpha \gamma_\rho \gamma_\beta q \right\rangle \right. \\
& + \left(-\frac{1}{18} - \frac{\epsilon}{24} \right) i g^2 \left\langle \bar{q} \overleftrightarrow{D}_\mu G_{\mu\alpha} G_{\nu\beta} \gamma_\alpha \gamma_\nu \gamma_\beta q \right\rangle + \left(-\frac{1}{18} - \frac{\epsilon}{24} \right) i g^2 \left\langle \bar{q} \overleftrightarrow{D}_\nu G_{\mu\alpha} G_{\nu\beta} \gamma_\alpha \gamma_\mu \gamma_\beta q \right\rangle \left. \right] \frac{1}{q^4} \\
& + q_\mu q_\nu \left[\left(\frac{1}{3} + \frac{\epsilon}{18} \right) i g^2 \left\langle \bar{q} \overleftrightarrow{D}_\rho G_{\mu\alpha} G_{\mu\beta} \gamma_\alpha \gamma_\rho \gamma_\beta q \right\rangle + \left(-\frac{1}{6} - \frac{5\epsilon}{72} \right) i g^2 \left\langle \bar{q} \overleftrightarrow{D}_\mu G_{\mu\alpha} G_{\nu\beta} \gamma_\alpha \gamma_\nu \gamma_\beta q \right\rangle \right. \\
& + \left. \left(-\frac{1}{6} - \frac{5\epsilon}{72} \right) i g^2 \left\langle \bar{q} \overleftrightarrow{D}_\nu G_{\mu\alpha} G_{\nu\beta} \gamma_\alpha \gamma_\mu \gamma_\beta q \right\rangle \right] \frac{1}{q^4},
\end{aligned} \tag{19}$$

where $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$, and the total result can be factorized as follows

$$\Pi_q^{d=8}(q^2) = (q_\mu q_\nu - q^2 g_{\mu\nu}) \left(-\frac{1}{24} g^3 \langle \bar{q}q \rangle \langle \bar{q}Gq \rangle \frac{1}{q^4} \right) + q_\mu q_\nu \left(-\frac{11}{27} g^3 \langle \bar{q}q \rangle \langle \bar{q}Gq \rangle \frac{1}{q^4} \right), \quad (20)$$

which is consistent with the factorized form in [10] (the condensate $\langle \bar{q}D_\rho G_{\mu\alpha} G_{\mu\beta} \gamma_\alpha \gamma_\rho \gamma_\beta q \rangle$ that cannot be factorized are set to $-\frac{7ig}{72} \langle \bar{q}q \rangle \langle \bar{q}Gq \rangle$ based on the formula $\langle \bar{q}D_\rho G_{\mu\alpha} G_{\mu\beta} \gamma_\alpha \gamma_\rho \gamma_\beta q \rangle = -\frac{7ig}{72} \langle \bar{q}q \rangle \langle \bar{q}Gq \rangle - \frac{1}{4} \langle \bar{q}d^{abc} D_\mu (G_{\alpha\rho}^a G_{\mu\beta}^b) T^c \gamma_\alpha \gamma_\rho \gamma_\beta q \rangle + \text{gluon condensates}$).

$$\begin{aligned} \pi_{\mu\nu}^{\text{VI}}(q) &= (q_\mu q_\nu - q^2 g_{\mu\nu}) \\ &\times \left[-\frac{1}{144\pi^2} g^4 O_1 - \frac{1}{144\pi^2} g^4 O_2 - \frac{1}{144\pi^2} g^4 O_3 + \frac{1}{16\pi^2} g^4 O_4 \right] \frac{1}{q^4} \\ &+ q_\mu q_\nu \left[-\frac{1}{48\pi^2} g^4 O_1 - \frac{1}{48\pi^2} g^4 O_2 - \frac{1}{48\pi^2} g^4 O_3 + \frac{1}{16\pi^2} g^4 O_4 \right] \frac{1}{q^4}, \end{aligned} \quad (21)$$

$$\begin{aligned} \pi_{\mu\nu}^{\text{VII}}(q) &= (q_\mu q_\nu - q^2 g_{\mu\nu}) \\ &\times \left[-\frac{1}{288\pi^2} g^4 O_1 + \frac{1}{288\pi^2} g^4 O_2 - \frac{1}{144\pi^2} g^4 O_3 + \frac{1}{144\pi^2} g^4 O_4 + \frac{7}{576\pi^2} g^3 O_8 \right] \frac{1}{q^4} \\ &+ q_\mu q_\nu \left[\frac{1}{96\pi^2} g^4 O_1 - \frac{1}{96\pi^2} g^4 O_2 - \frac{1}{16\pi^2} g^4 O_3 + \frac{1}{16\pi^2} g^4 O_4 + \frac{1}{192\pi^2} g^3 O_8 \right] \frac{1}{q^4}, \end{aligned} \quad (22)$$

$$\begin{aligned} \pi_{\mu\nu}^{\text{VIII}}(q) &= (q_\mu q_\nu - q^2 g_{\mu\nu}) \\ &\times \left[-\frac{1}{288\pi^2} g^4 O_1 + \frac{1}{288\pi^2} g^4 O_2 + \frac{1}{54\pi^2} g^4 O_3 - \frac{1}{54\pi^2} g^4 O_4 \right. \\ &\quad \left. - \frac{1}{864\pi^2} g^3 O_5 + \frac{1}{288\pi^2} g^2 O_7 - \frac{5}{864\pi^2} g^3 O_8 \right] \frac{1}{q^4} \\ &+ q_\mu q_\nu \left[\frac{1}{576\pi^2} g^4 O_1 - \frac{1}{576\pi^2} g^4 O_2 + \frac{7}{288\pi^2} g^4 O_3 - \frac{7}{288\pi^2} g^4 O_4 \right. \\ &\quad \left. - \frac{1}{576\pi^2} g^3 O_5 + \frac{1}{192\pi^2} g^2 O_7 - \frac{1}{288\pi^2} g^3 O_8 \right] \frac{1}{q^4}, \end{aligned} \quad (23)$$

$$\begin{aligned} \pi_{\mu\nu}^{\text{IX}}(q) &= (q_\mu q_\nu - q^2 g_{\mu\nu}) \\ &\times \left[\left(-\frac{1}{72\pi^2} \frac{1}{\hat{\epsilon}} + \frac{11}{864\pi^2} \right) g^4 O_1 + \left(\frac{1}{216\pi^2} \frac{1}{\hat{\epsilon}} + \frac{5}{864\pi^2} \right) g^4 O_2 \right. \\ &\quad \left. + \left(\frac{5}{72\pi^2} \frac{1}{\hat{\epsilon}} - \frac{19}{864\pi^2} \right) g^4 O_3 + \left(-\frac{7}{216\pi^2} \frac{1}{\hat{\epsilon}} - \frac{53}{864\pi^2} \right) g^4 O_4 \right] \frac{1}{q^4} \\ &+ q_\mu q_\nu \left[\left(\frac{1}{24\pi^2} \frac{1}{\hat{\epsilon}} + \frac{13}{288\pi^2} \right) g^4 O_1 + \left(\frac{1}{72\pi^2} \frac{1}{\hat{\epsilon}} + \frac{11}{864\pi^2} \right) g^4 O_2 \right. \\ &\quad \left. + \left(-\frac{1}{12\pi^2} \frac{1}{\hat{\epsilon}} + \frac{1}{72\pi^2} \right) g^4 O_3 + \left(-\frac{5}{36\pi^2} \frac{1}{\hat{\epsilon}} - \frac{41}{216\pi^2} \right) g^4 O_4 \right] \frac{1}{q^4}, \end{aligned} \quad (24)$$

$$\begin{aligned} \pi_{\mu\nu}^{\text{X}}(q) &= (q_\mu q_\nu - q^2 g_{\mu\nu}) \\ &\times \left[\left(\frac{1}{72\pi^2} \frac{1}{\hat{\epsilon}} - \frac{1}{288\pi^2} \right) g^4 O_3 + \left(-\frac{1}{72\pi^2} \frac{1}{\hat{\epsilon}} + \frac{1}{288\pi^2} \right) g^4 O_4 + \left(-\frac{1}{144\pi^2} \frac{1}{\hat{\epsilon}} + \frac{1}{576\pi^2} \right) g^3 O_8 \right] \frac{1}{q^4} \\ &+ q_\mu q_\nu \left(\frac{1}{48\pi^2} g^4 O_3 - \frac{1}{48\pi^2} g^4 O_4 - \frac{1}{96\pi^2} g^3 O_8 \right) \frac{1}{q^4}, \end{aligned} \quad (25)$$

$$\begin{aligned}
\pi_{\mu\nu}^{\text{XI}}(q) = & (q_\mu q_\nu - q^2 g_{\mu\nu}) \\
& \times \left[-\frac{1}{432\pi^2} g^4 O_1 + \frac{1}{432\pi^2} g^4 O_2 - \frac{1}{18\pi^2} \frac{1}{\hat{\epsilon}} g^4 O_3 + \frac{1}{18\pi^2} \frac{1}{\hat{\epsilon}} g^4 O_4 \right. \\
& + \left(\frac{1}{108\pi^2} \frac{1}{\hat{\epsilon}} + \frac{1}{108\pi^2} \right) g^3 O_5 + \left(\frac{1}{144\pi^2} \frac{1}{\hat{\epsilon}} - \frac{1}{216\pi^2} \right) g^3 O_8 \Big] \frac{1}{q^4} \\
& + q_\mu q_\nu \left[-\frac{1}{72\pi^2} g^4 O_1 + \frac{1}{72\pi^2} g^4 O_2 - \frac{1}{24\pi^2} g^4 O_3 \right. \\
& + \left. \frac{1}{24\pi^2} g^4 O_4 + \left(\frac{1}{36\pi^2} \frac{1}{\hat{\epsilon}} + \frac{7}{216\pi^2} \right) g^3 O_5 + \frac{5}{576\pi^2} g^3 O_8 \right] \frac{1}{q^4},
\end{aligned} \tag{26}$$

where $O_1 = \text{Tr}(G_{\mu\nu} G_{\mu\nu} G_{\alpha\beta} G_{\alpha\beta})$, $O_2 = \text{Tr}(G_{\mu\nu} G_{\alpha\beta} G_{\mu\nu} G_{\alpha\beta})$, $O_3 = \text{Tr}(G_{\mu\nu} G_{\nu\alpha} G_{\alpha\beta} G_{\beta\mu})$, $O_4 = \text{Tr}(G_{\mu\nu} G_{\alpha\beta} G_{\nu\alpha} G_{\beta\mu})$, $O_5 = f_{abc} G_{\mu\nu}^a j_\mu^b j_\nu^c$, $O_6 = f_{abc} G_{\mu\nu}^a j_\lambda^b D_\lambda G_{\mu\nu}^c$, $O_7 = j_\mu^a D^2 j_\mu^a$, $O_8 = f_{abc} G_{\mu\nu}^a G_{\nu\lambda}^b D^2 G_{\lambda\mu}^c$, and $\frac{1}{\hat{\epsilon}} = \frac{1}{\epsilon} - \frac{1}{2} \ln \frac{-q^2}{\mu^2} + \frac{1}{2} \ln 4\pi - \frac{\gamma_E}{2}$.

Appendix B: Results of Numerical Analysis

HDC /OPE	<10%	<5%	<1%	<15% (d≤6)	- (d≤6)
$[\tau_{\min}, \tau_{\max}]/\text{GeV}^{-2}$	[0.24,0.89]	[0.26,0.81]	[0.32,0.60]	[0.36, 0.45]	[0.32,0.60]
s_0/GeV^2	$10.73^{+0.78}_{-0.65}$	$9.70^{+0.63}_{-0.55}$	$7.92^{+0.39}_{-0.37}$	$7.01^{+0.35}_{-0.34}$	$7.64^{+0.45}_{-0.42}$
m_H/GeV	$2.34^{+0.06}_{-0.05}$	$2.29^{+0.05}_{-0.05}$	$2.16^{+0.05}_{-0.04}$	$2.08^{+0.06}_{-0.06}$	$2.13^{+0.05}_{-0.05}$
$f_H^2/10^{-3}\text{GeV}^2$	$0.80^{+0.05}_{-0.05}$	$0.74^{+0.05}_{-0.05}$	$0.63^{+0.04}_{-0.04}$	$0.57^{+0.04}_{-0.03}$	$0.62^{+0.04}_{-0.04}$

TABLE IV: Matching results with input parameters in Set I (10% uncertainties for input phenomenological parameters, $k_1 = k_2 = 1$).

HDC /OPE	<10%	<5%	<2%	<15% (d≤6)	- (d≤6)
$[\tau_{\min}, \tau_{\max}]/\text{GeV}^{-2}$	[0.23,0.69]	[0.25,0.61]	[0.28,0.51]	[0.32, 0.38]	[0.28,0.51]
s_0/GeV^2	$11.47^{+0.88}_{-0.74}$	$10.40^{+0.69}_{-0.61}$	$9.38^{+0.52}_{-0.48}$	$8.10^{+0.43}_{-0.40}$	$8.75^{+0.56}_{-0.50}$
m_H/GeV	$2.50^{+0.07}_{-0.06}$	$2.43^{+0.06}_{-0.06}$	$2.36^{+0.06}_{-0.06}$	$2.25^{+0.06}_{-0.06}$	$2.30^{+0.06}_{-0.06}$
$f_H^2/10^{-3}\text{GeV}^2$	$0.78^{+0.04}_{-0.04}$	$0.72^{+0.04}_{-0.04}$	$0.66^{+0.04}_{-0.04}$	$0.58^{+0.04}_{-0.03}$	$0.62^{+0.04}_{-0.03}$

TABLE V: Matching results with input parameters in Set I (10% uncertainties for input phenomenological parameters, $k_1 = k_2 = 2$).

HDC /OPE	<10%	<5%	<2%	<15% (d≤6)	-(d≤6)
$[\tau_{\min}, \tau_{\max}]/\text{GeV}^{-2}$	[0.21,0.60]	[0.23,0.53]	[0.25,0.44]	[0.29, 0.34]	[0.25,0.44]
s_0/GeV^2	$12.66^{+1.07}_{-0.87}$	$11.54^{+0.85}_{-0.72}$	$10.45^{+0.64}_{-0.57}$	$9.12^{+0.53}_{-0.49}$	$9.65^{+0.65}_{-0.58}$
m_H/GeV	$2.65^{+0.08}_{-0.07}$	$2.58^{+0.07}_{-0.07}$	$2.51^{+0.07}_{-0.06}$	$2.39^{+0.07}_{-0.06}$	$2.44^{+0.07}_{-0.07}$
$f_H^2/10^{-3}\text{GeV}^2$	$0.79^{+0.04}_{-0.04}$	$0.73^{+0.04}_{-0.04}$	$0.67^{+0.04}_{-0.04}$	$0.60^{+0.03}_{-0.03}$	$0.63^{+0.03}_{-0.03}$

TABLE VI: Matching results with input parameters in Set I (10% uncertainties for input phenomenological parameters, $k_1 = k_2 = 3$).

$ \text{HDC} /\text{OPE}$	$<10\%$	$<5\%$	$<3\%$
$[\tau_{\min}, \tau_{\max}]/\text{GeV}^{-2}$	[0.21,0.53]	[0.23,0.46]	[0.25,0.41]
s_0/GeV^2	$12.23^{+0.85}_{-0.73}$	$11.17^{+0.67}_{-0.60}$	$10.66^{+0.58}_{-0.53}$
m_H/GeV	$2.64^{+0.07}_{-0.07}$	$2.58^{+0.06}_{-0.06}$	$2.54^{+0.06}_{-0.06}$
$f_H^2/10^{-3}\text{GeV}^2$	$0.76^{+0.04}_{-0.04}$	$0.71^{+0.04}_{-0.04}$	$0.68^{+0.04}_{-0.04}$

TABLE VII: Matching results with input parameters in Set I (10% uncertainties for input phenomenological parameters, $k_1 = 3$, $k_2 = 5$).

$ \text{HDC} /\text{OPE}$	$<10\%$	$<7\%$	$<5\%$	$<15\% (d \leq 6)$	- ($d \leq 6$)
$[\tau_{\min}, \tau_{\max}]/\text{GeV}^{-2}$	[0.36,0.81]	[0.38,0.74]	[0.39,0.68]	[0.48,0.59]	[0.39,0.68]
s_0/GeV^2	$6.98^{+0.37}_{-0.40}$	$6.62^{+0.33}_{-0.36}$	$6.32^{+0.30}_{-0.33}$	$5.12^{+0.28}_{-0.33}$	$5.17^{+0.29}_{-0.35}$
m_H/GeV	$1.98^{+0.04}_{-0.05}$	$1.95^{+0.04}_{-0.05}$	$1.93^{+0.04}_{-0.05}$	$1.77^{+0.04}_{-0.05}$	$1.78^{+0.04}_{-0.05}$
$f_H^2/10^{-3}\text{GeV}^2$	$0.65^{+0.05}_{-0.04}$	$0.62^{+0.05}_{-0.04}$	$0.60^{+0.04}_{-0.04}$	$0.53^{+0.04}_{-0.03}$	$0.54^{+0.04}_{-0.04}$

TABLE VIII: Matching results with input parameters in Set II (10% uncertainties for input phenomenological parameters, $k_1 = k_2 = 1$).

$ \text{HDC} /\text{OPE}$	$<10\%$	$<7\%$	$<4\%$	$<15\% (d \leq 6)$	- ($d \leq 6$)
$[\tau_{\min}, \tau_{\max}]/\text{GeV}^{-2}$	[0.29,0.66]	[0.30,0.61]	[0.32,0.54]	[0.39,0.45]	[0.32,0.54]
s_0/GeV^2	$8.85^{+0.57}_{-0.53}$	$8.41^{+0.50}_{-0.47}$	$7.97^{+0.43}_{-0.42}$	$6.57^{+0.41}_{-0.40}$	$6.70^{+0.43}_{-0.42}$
m_H/GeV	$2.25^{+0.06}_{-0.06}$	$2.22^{+0.05}_{-0.06}$	$2.18^{+0.05}_{-0.05}$	$2.02^{+0.06}_{-0.06}$	$2.04^{+0.06}_{-0.06}$
$f_H^2/10^{-3}\text{GeV}^2$	$0.67^{+0.04}_{-0.04}$	$0.65^{+0.04}_{-0.04}$	$0.62^{+0.04}_{-0.04}$	$0.54^{+0.04}_{-0.03}$	$0.55^{+0.04}_{-0.03}$

TABLE IX: Matching results with input parameters in Set II (10% uncertainties for input phenomenological parameters, $k_1 = k_2 = 2$).

$ \text{HDC} /\text{OPE}$	$<10\%$	$<7\%$	$<4\%$	$<15\% (d \leq 6)$	- ($d \leq 6$)
$[\tau_{\min}, \tau_{\max}]/\text{GeV}^{-2}$	[0.25,0.59]	[0.26,0.54]	[0.27,0.48]	[0.33,0.38]	[0.27,0.48]
s_0/GeV^2	$10.57^{+0.82}_{-0.72}$	$10.01^{+0.70}_{-0.64}$	$9.41^{+0.59}_{-0.55}$	$7.82^{+0.51}_{-0.50}$	$8.09^{+0.58}_{-0.55}$
m_H/GeV	$2.46^{+0.07}_{-0.07}$	$2.42^{+0.07}_{-0.07}$	$2.38^{+0.06}_{-0.07}$	$2.22^{+0.07}_{-0.07}$	$2.24^{+0.07}_{-0.07}$
$f_H^2/10^{-3}\text{GeV}^2$	$0.71^{+0.05}_{-0.04}$	$0.68^{+0.04}_{-0.04}$	$0.65^{+0.04}_{-0.04}$	$0.56^{+0.04}_{-0.03}$	$0.58^{+0.04}_{-0.03}$

TABLE X: Matching results with input parameters in Set II (10% uncertainties for input phenomenological parameters, $k_1 = k_2 = 3$).

$ \text{HDC} /\text{OPE}$	$<10\%$	$<7\%$	$<5\%$
$[\tau_{\min}, \tau_{\max}]/\text{GeV}^{-2}$	[0.24,0.53]	[0.25,0.49]	[0.26,0.45]
s_0/GeV^2	$10.59^{+0.69}_{-0.63}$	$10.15^{+0.62}_{-0.57}$	$9.76^{+0.55}_{-0.52}$
m_H/GeV	$2.49^{+0.07}_{-0.07}$	$2.46^{+0.06}_{-0.06}$	$2.43^{+0.06}_{-0.06}$
$f_H^2/10^{-3}\text{GeV}^2$	$0.71^{+0.04}_{-0.04}$	$0.68^{+0.04}_{-0.04}$	$0.66^{+0.04}_{-0.04}$

TABLE XI: Matching results with input parameters in Set II (10% uncertainties for input phenomenological parameters, $k_1 = 3$, $k_2 = 5$).

-
- [1] K. A. Olive *et al.* [Particle Data Group Collaboration], *Chin. Phys. C* **38**, 090001 (2014).
 - [2] R. Zhang, Y. -B. Ding, X. -Q. Li and P. R. Page, *Phys. Rev. D* **65**, 096005 (2002) [hep-ph/0111361].
 - [3] Z. F. Zhang and H. Y. Jin, *Phys. Rev. D* **71**, 011502 (2005) [hep-ph/0412226].
 - [4] I. J. General, P. Wang, S. R. Cotanch and F. J. Llanes-Estrada, *Phys. Lett. B* **653**, 216 (2007) [arXiv:0707.1286 [hep-ph]].
 - [5] H. -X. Chen, A. Hosaka and S. -L. Zhu, *Phys. Rev. D* **78**, 054017 (2008) [arXiv:0806.1998 [hep-ph]].
 - [6] M. S. Chanowitz and S. R. Sharpe, *Nucl. Phys. B* **222**, 211 (1983) [Erratum-ibid. B **228**, 588 (1983)]; T. Barnes, F. E. Close, F. de Viron and J. Weyers, *Nucl. Phys. B* **224**, 241 (1983).
 - [7] N. Isgur and J. E. Paton, *Phys. Rev. D* **31**, 2910 (1985); T. Barnes, F. E. Close and E. S. Swanson, *Phys. Rev. D* **52**, 5242 (1995) [hep-ph/9501405].
 - [8] C. Michael, [hep-lat/0302001]; C. McNeile *et al.* [UKQCD Collaboration], *Phys. Rev. D* **73**, 074506 (2006) [hep-lat/0603007]; J. J. Dudek, *Phys. Rev. D* **84**, 074023 (2011) [arXiv:1106.5515 [hep-ph]].
 - [9] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, *Nucl. Phys. B* **147**, 385 (1979).
 - [10] I. I. Balitsky, D. Diakonov and A. V. Yung, *Phys. Lett. B* **112**, 71 (1982).
 - [11] J. Govaerts, F. de Viron, D. Gusbin and J. Weyers, *Phys. Lett. B* **128**, 262 (1983).
 - [12] I. I. Balitsky, D. Diakonov and A. V. Yung, *Z. Phys. C* **33**, 265 (1986); J. Govaerts, F. de Viron, D. Gusbin and J. Weyers, *Nucl. Phys. B* **248**, 1 (1984); J. I. Latorre, P. Pascual and S. Narison, *Z. Phys. C* **34**, 347 (1987); J. Govaerts, L. J. Reinders, P. Francken, X. Gonze and J. Weyers, *Nucl. Phys. B* **284**, 674 (1987); S. Narison, *Nucl. Phys. A* **675**, 54C (2000) [hep-ph/9909470];
 - [13] K. G. Chetyrkin and S. Narison, *Phys. Lett. B* **485**, 145 (2000) [hep-ph/0003151].
 - [14] H. Y. Jin and J. G. Korner, *Phys. Rev. D* **64**, 074002 (2001) [hep-ph/0003202].
 - [15] H. Y. Jin, J. G. Korner and T. G. Steele, *Phys. Rev. D* **67**, 014025 (2003) [hep-ph/0211304].
 - [16] S. Narison, *Phys. Lett. B* **675**, 319 (2009) [arXiv:0903.2266 [hep-ph]].
 - [17] Z. F. Zhang, H. Y. Jin and T. G. Steele, *Chin. Phys. Lett.* **31**, 051201 (2014) [arXiv:1312.5432 [hep-ph]].
 - [18] D. B. Leinweber, *Annals Phys.* **254**, 328 (1997) [nucl-th/9510051].
 - [19] F. X. Lee, *Phys. Rev. C* **57**, 322 (1998) [hep-ph/9707332].
 - [20] F. X. Lee, D. B. Leinweber and X. -M. Jin, *Phys. Rev. D* **55**, 4066 (1997) [nucl-th/9611011].
 - [21] F. X. Lee, *Phys. Lett. B* **419**, 14 (1998) [hep-ph/9707411].
 - [22] L. Wang and F. X. Lee, *Phys. Rev. D* **78**, 013003 (2008) [arXiv:0804.1779 [hep-ph]].
 - [23] D. J. Broadhurst and S. C. Generalis, *Phys. Lett. B* **165**, 175 (1985).
 - [24] A. G. Grozin, *Int. J. Mod. Phys. A* **10**, 3497 (1995) [hep-ph/9412238].
 - [25] V. A. Novikov, M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, *Fortsch. Phys.* **32**, 585 (1984).
 - [26] S. Narison, *Phys. Lett. B* **624**, 223 (2005) [hep-ph/0412152].
 - [27] R. A. Bertlmann, G. Launer and E. de Rafael, *Nucl. Phys. B* **250**, 61 (1985).
 - [28] R. A. Bertlmann, C. A. Dominguez, M. Loewe, M. Perrottet and E. de Rafael, *Z. Phys. C* **39**, 231 (1988).
 - [29] G. Launer, S. Narison and R. Tarrach, *Z. Phys. C* **26**, 433 (1984).
 - [30] S. Narison, *Phys. Lett. B* **300**, 293 (1993).
 - [31] S. Narison, *Phys. Lett. B* **361**, 121 (1995) [hep-ph/9504334].
 - [32] S. Narison, *Phys. Lett. B* **673**, 30 (2009) [arXiv:0901.3823 [hep-ph]].
 - [33] V. A. Novikov, M. A. Shifman, A. I. Vainshtein, M. B. Voloshin and V. I. Zakharov, *Nucl. Phys. B* **237**, 525 (1984).
 - [34] E. Bagan, J. I. Latorre, P. Pascual and R. Tarrach, *Nucl. Phys. B* **254**, 555 (1985).
 - [35] S. N. Nikolaev and A. V. Radyushkin, *Phys. Lett. B* **124**, 243 (1983).
 - [36] S. Narison, *Phys. Lett. B* **706**, 412 (2012) [arXiv:1105.2922 [hep-ph]]; S. Narison, *Phys. Lett. B* **707**, 259 (2012) [arXiv:1105.5070 [hep-ph]].
 - [37] S. Narison, arXiv:1409.8148.
 - [38] V. A. Novikov, M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, *Nucl. Phys. B* **191**, 301 (1981).
 - [39] M. D'Elia, A. Di Giacomo and E. Meggiolaro, *Phys. Lett. B* **408**, 315 (1997) [hep-lat/9705032].
 - [40] S. Narison and E. de Rafael, *Phys. Lett. B* **103**, 57 (1981).
 - [41] Stephan Narison, "QCD as a theory of hadrons: From Partons to Confinement", Cambridge Univ. Press, 2004.
 - [42] K. G. Chetyrkin, S. Narison and V. I. Zakharov, *Nucl. Phys. B* **550**, 353 (1999) [hep-ph/9811275].
 - [43] S. Narison and V. I. Zakharov, *Phys. Lett. B* **522**, 266 (2001) [hep-ph/0110141].
 - [44] S. Narison and V. I. Zakharov, *Phys. Lett. B* **679**, 355 (2009) [arXiv:0906.4312 [hep-ph]].